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Radiation from an Array of Gray Circular Fins of Trapezoidal Profile

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A numerical method is developed to calculate the temperature distribution and radiation heat transfer for an annular fin and tube radiator, with fins having trapezoidal profiles. All surfaces are assumed gray and to emit and reflect diffusely. Radiative interactions between adjacent fins and between the fins and tube are included. The thermal conductivity of the fin material may vary linearly with temperature. The independent dimensionless parameters and their ranges are ε = 0.8–1.0, r_i/r_o = 0.20–0.67, L/r_o = 0.25–2.0, and N_C = 0–0.8. Results of a parametric study of the special case of circular fins of triangular profile having constant thermal conductivity are presented, and used to optimize a fin array with respect to minimum weight.

Nomenclature

B_i = radiosity of surface i
F = angle factor
k = thermal conductivity
k_0 = thermal conductivity at T_0
L = center to center spacing between fins
N = number of node points
q_r = radiant heat flux
Q = heat transfer from the fin array
Q_k = conduction heat transfer
Q_R = radiant heat transfer
r = radial distance
r_i = tube radius
r_o = fin outside radius
t = local fin thickness
t_i = thickness of fin at $r = r_i$
t_o = thickness of fin at $r = r_o$
T = temperature
T_0 = base temperature
ε = emissivity
Λ = matrix defined by Eq. (9)
σ = Stefan-Boltzmann constant
χ = matrix defined by Eq. (6)
ψ = inverse of the χ matrix
Ω = vector defined by Eq. (7)
C = k/k_0
C' = $(T_0/k_0)(dk/dT)$
N_C = $2\sigma\varepsilon T_0^3 r_i^2/k_0 t_i$, the conduction number
N_L = L/r_o
N_R = r_i/r_o
N_t = t_o/t_i
Q^* = $Q/(2\pi r_i L\sigma T_0^4)$
X = r/r_o
Y = t/t_i
θ = T/T_0

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Introduction

FINNED surfaces have been widely used for rejection of heat in space applications. Fin radiation has, therefore, been extensively studied during the past two decades. The original work was carried out for the case of one-dimensional fins which did not radiatively interact with other surfaces.[1-6] More recent work has included radiative interactions for some of the simpler geometries.[7-13]

One geometry which has received some attention is the annular fin and tube radiator in Fig. 1. This system has been analyzed for the case of black circular fins of rectangular profile, including the fin-to-fin and fin-to-tube interactions.[9] A variable area profile (e.g., trapezoidal or triangular) would, however, be more effective than a rectangular profile, on the basis of heat transfer per unit weight.

The purpose of the work reported here is to develop a numerical method to analyze an annular fin and tube array having circular fins of trapezoidal profile. The method is then used to make a parametric study of this type of fin array, to provide information for optimization of the design with respect to weight. The surfaces are assumed to be gray and to emit and reflect diffusely. Radiative interactions among all elements of the array are included.

Statement of the Problem

A section of the fin array to be analyzed is shown in Fig. 2. The purpose of the analysis is to determine the temperature distribution in the fins and to calculate the heat transfer from the fin array. The analysis is based on the following assumptions: 1) steady-state operation; 2) the temperature distribution in the fin is one dimensional; 3) the tube surface is isothermal; 4) all surfaces are radiatively gray and diffuse and have the same emissivity; 5) there is no incident radiation from external sources; 6) thermal conductivity is a linear function of temperature; 7) the fin thickness at the base t_i is small compared to the spacing between the center planes of adjacent fins, L. The midplane of the fin (line a-a, Fig. 2) is a symmetry plane, so that the segment studied has a thickness one-half that of the fin.

The conduction equation may be derived by making a steady-state energy balance on a typical element of thickness dr as follows.

$$Q_{k,\mathrm{IN}} = -k\pi rt\, dT/dr$$

$$Q_{k,\mathrm{OUT}} = -k\pi rt\frac{dT}{dr} + \frac{d}{dr}\left(-k\pi rt\frac{dT}{dr}\right)dr$$

$$Q_{R,\mathrm{OUT}} = q_r 2\pi r\, dr$$

Note that $k = k(T)$, so that $dk/dr = (dk/dT)(dT/dr)$. Then

$$\frac{d^2T}{dr^2} + \frac{1}{t}\left(\frac{dt}{dr} + \frac{t}{r}\right)\frac{dT}{dr} + \frac{1}{k}\frac{dk}{dT}\left(\frac{dT}{dr}\right)^2 = \frac{2q_r}{kt} \tag{1}$$

The boundary conditions are

$$T = T_0 \quad \text{at} \quad r = r_i$$

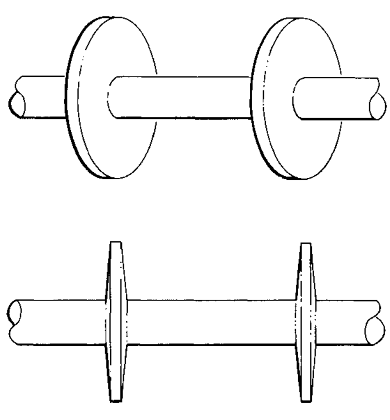

Fig. 1 Schematic of the physical system.

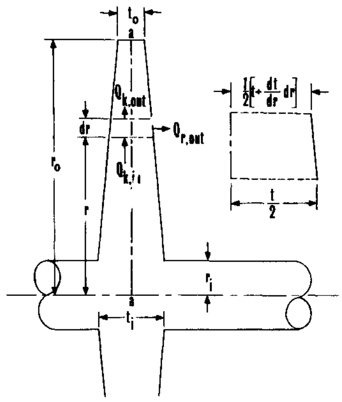

Fig. 2 Section of fin for conduction analysis.

and

$$dT/dr = -\sigma\varepsilon T^4/k \quad \text{at} \quad r = r_o$$

The second boundary condition takes into account radiation loss from the end of the fin.

Introducing the following nondimensional parameters

$$X = r/r_o \qquad Y = t/t_i$$

$$C = k/k_0 \qquad \theta = T/T_0$$

$$N_R = r_i/r_o \qquad N_t = t_o/t_i$$

$$N_C = 2\sigma\varepsilon T_0{}^3 r_i{}^2/k_0 t_i$$

and since conductivity varies linearly with temperature

$$C = 1 + C'(\theta - 1)$$

Eq. (1) in nondimensional form is

$$\frac{d^2\theta}{dX^2} + \frac{1}{Y}\left(\frac{dY}{dX} + \frac{Y}{X}\right)\frac{d\theta}{dX} + \frac{C'}{[1+C'(\theta-1)]}\left(\frac{d\theta}{dX}\right)^2 = \frac{N_C(q_r/\sigma T_0{}^4)}{\varepsilon N_R{}^2 Y[1+C'(\theta-1)]} \tag{2}$$

For a trapezoidal profile

$$t = t_i + (r - r_i)(t_o - t_i)/(r_o - r_i)$$

Then

$$Y = 1 + (X - N_R)(N_t - 1)/(1 - N_R)$$

and

$$dY/dX = (N_t - 1)/(1 - N_R)$$

The boundary conditions are

$$\theta = 1 \quad \text{at} \quad X = N_R \tag{3}$$

and

$$\frac{d\theta}{dX} = -\frac{\sigma\varepsilon T_0{}^3 r_0}{k_0[1+C'(\theta-1)]}\theta^4 \quad \text{at} \quad X = 1 \tag{4}$$

The relationship between q_r and T must be obtained by solving the problem of radiation within a nonisothermal gray enclosure. Since a numerical solution will be used, it is convenient to immediately use the finite-difference approximation, and assume that each strip of exposed surface having width dr and circumference $2\pi r$ has a uniform temperature T_j, where $1 \leq j \leq N$ and N is the number of strips. The applicable radiation equations are the following.[14]

$$\sum_{j=1}^{N+1} \chi_{ij} B_j = \Omega_i \tag{5}$$

where

$$\chi_{ij} = \frac{\delta_{ij} - (1-\varepsilon_i)F_{A_i - A_j}}{\varepsilon_i} \tag{6}$$

$$\Omega_j = \sigma T_j^4 \tag{7}$$

and

$$\delta_{ij} = \begin{matrix} 0 & i \neq j \\ 1 & i = j \end{matrix}$$

Note that the isothermal tube surface is also included, giving rise to $(N+1)$ equations. The local surface heat flux may be computed from

$$q_{r_i} = \sum_{j=1}^{N+1} \Lambda_{ij}\Omega_j$$

or in dimensionless form

$$\frac{q_{r_i}}{\sigma T_0^4} = \sum_{j=1}^{N+1} \Lambda_{ij}\theta_j^4 \tag{8}$$

where

$$\Lambda_{ij} = [\varepsilon_i/(1-\varepsilon_i)](\delta_{ij} - \psi_{ij}) \tag{9}$$

and

$$[\psi] = [\chi]^{-1}$$

The simultaneous solution of Eqs. (2–9) constitutes the solution of the problem.

Note that the independent dimensionless parameters which must be specified to define a particular case include N_R, N_t, N_C, ε, and C'. The angle factors in Eq. (6) also depend on N_L where

$$N_L = L/r_o$$

Method of Solution

The numerical method used to obtain the temperature distribution and heat flux in the fin array is summarized in the flow diagram shown in Fig. 3. The parameters N_R, N_L, and N_t specify the geometry. In addition, ε, C', and N_C are specified. A subroutine is then called which calculates all angle factors using the contour integral method discussed in Ref. 14. Then the $[\chi]$ matrix is calculated using Eq. (6) and is inverted to form the $[\psi]$ matrix. Note that if emissivity is assumed constant, the inversion is done only once since the terms are not temperature dependent. The $[\Lambda]$ matrix may then be calculated from Eq. (9).

An initial assumption of the temperature distribution is now made and an iterative solution proceeds as follows.

1) The local radiant heat flux (based on the assumed temperature distribution) is calculated from Eqs. (7) and (8).

2) The total heat transfer from the fin is computed by summing the local fluxes multiplied by their respective areas, and adding the radiative loss at the end of the fin.

3) The radiative heat transfer is equated to conduction at the base to determine $d\theta/dX$ at the fin base (first node point).

4) The Runge-Kutta-Nyström method[15] is then used along with the local radiant heat fluxes obtained in step 3 to calculate a new temperature distribution.

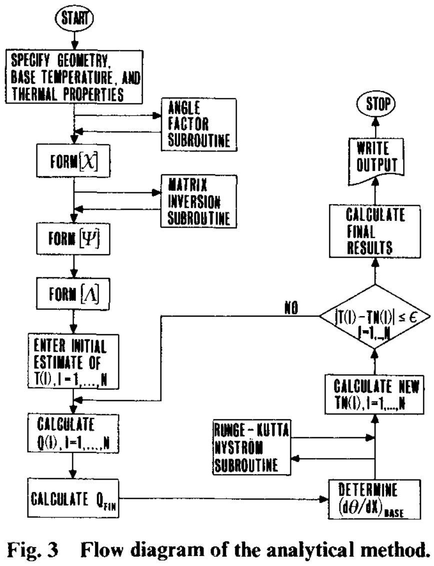

Fig. 3 Flow diagram of the analytical method.

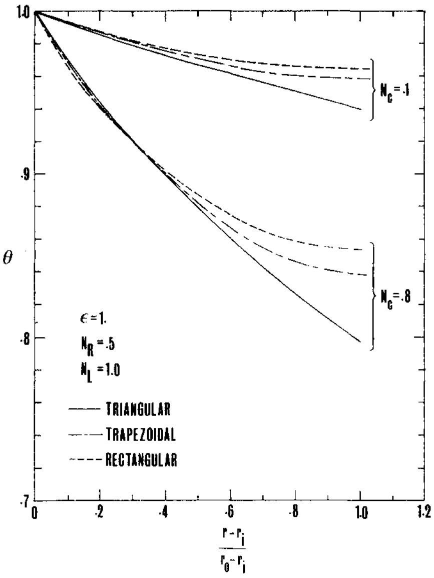

Fig. 4 Comparison of temperature distribution for rectangular, trapezoidal, and triangular profiles.

5) If the distribution is not identical to the assumed distribution within a set tolerance, steps 1–4 are repeated until convergence is obtained.

6) Final results, including total heat flux from the fin array, are then computed.

The method just discussed had to be damped to insure convergence. The dimensionless temperature at a point was not permitted to change to the new value θ_{NEW}, calculated in step 4, but instead was set equal to $\theta + D(\theta_{NEW} - \theta)$, where θ was the previous dimensionless temperature at that point and D was the damping factor. A value of $D = 0.2$ led to fairly rapid convergence. The criterion for convergence was that the change in dimensionless temperature at all node points be less than 0.01% for successive iterations.

The number of node points necessary to obtain good accuracy was determined by systematically varying N and comparing results. It was found that 5 divisions gave a value of dimensionless heat transfer within 1% of that computed using 15 divisions. Eight divisions were used, however, which resulted in somewhat better accuracy (0.3%) without requiring excessive computer time.

Initial runs were made for the case of $\varepsilon = 1$ and fins of rectangular profile, since results of these runs could be compared to results in Ref. 8. Agreement was excellent for all values of N_R, N_L, and N_C. An additional independent check was made for all data computed at $N_C = 0$. This corresponds to fins of infinite thermal conductivity, resulting in an isothermal fin array. That solution which was easily obtained by direct computation was in excellent agreement with results obtained from the numerical method for all values of ε. Finally, a check was made of the accuracy of the matrix inversion. The product of the $[\chi]$ matrix and its inverse was compared to the identity matrix. Discrepancies were less than 1 part in 10^8 for all terms using single precision. It was therefore inferred that computation errors were insignificant.

Results and Discussion

The numerical method developed here is capable of predicting the fin temperature distribution and heat flux from the system composed of tube and fins, for any specified values of N_L, N_R, N_C, ε, and dk/dT, and for any trapezoidal profile from rectangular to triangular.

Using the numerical method, a parametric study was conducted. First, a series of three runs was conducted to compare

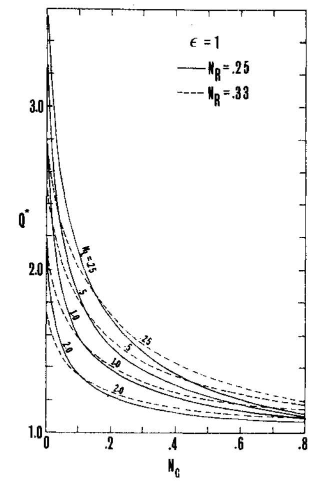

Fig. 5 Dimensionless heat flux from the fin array; $\varepsilon = 1.0$, $N_R = 0.25$, and 0.333.

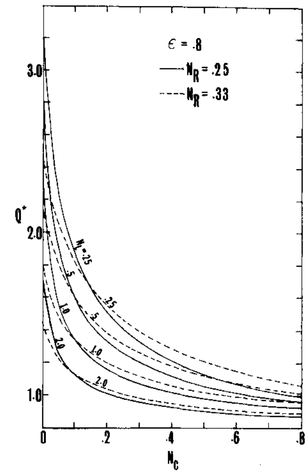

Fig. 7 Dimensionless heat flux from the fin array; $\varepsilon = 0.8$, $N_R = 0.25$, and 0.333.

fins of rectangular, trapezoidal ($N_t = 0.5$), and triangular profile. The resulting temperature distributions and heat flux parameters are shown in Fig. 4. The intersection of the temperature profiles is, perhaps, unexpected. This same behavior was reported by Karlekar and Chao[7] for the case of straight fins of rectangular and triangular profiles. It should also be noted that the temperature gradient does not go to zero at the fin tip. The assumption of zero temperature gradient at the tip is used in the Ref. 7 analysis. This is apparently a result of the authors' assumption of insulated ends for trapezoidal fins being retained as the trapezoidal shape approached the triangular case. Under the more realistic end condition used here, the temperature gradient does not vanish as t_o approaches zero.

The most important result of the profile comparison, however, was the fact that the heat transfer from the array having fins of triangular profile was within a few percent of the heat transfer from the fin array having fins of rectangular profile. Since the rectangular fins weigh more than twice as much as the triangular ones, the latter are clearly superior on the basis of heat transfer per unit weight. Therefore, extensive results are presented only for circular fins of triangular profile. All further discussion is limited to the triangular profile.

After the profile comparison was completed, the effects of variable thermal conductivity were considered. This was done by solving several cases for a range of values of N_C using the thermal conductivity vs temperature data of 18-8 stainless steel (type 304). The results were compared to those of cases which assumed constant thermal conductivity. It was found that for $0 \leqq N_C \leqq 0.8$, the error caused by using the constant conductivity results with k evaluated at T_0, was less than 3%. Copper and aluminum exhibit even less variation of k with T, so that the constant conductivity results may be used for those materials with confidence. The effect of variable conductivity may be significant for a few specialized materials at high temperature.[1]

Therefore, the subsequent results presented here are for the case of constant thermal conductivity.

The remaining independent variables in the analysis are N_R, N_L, N_C, and ε. Since high emissivity values are desirable in practice, results are obtained only for $\varepsilon = 0.8$, 0.9, and 1.0. The parameters N_R, N_L, and N_C are varied over ranges which should include all practical applications. A total of 480 runs was made. Results are summarized in Figs. 5–8 in the form of the dimensionless parameter Q^*, which is defined as the ratio of radiation from the fin array (including the base) to the radiation which would be emitted by a black unfinned tube at the same temperature.

The effect of emissivity, is a decrease in Q^* with decreasing ε as anticipated. However, the percent decrease in heat transfer is not as large as that caused by an equal decrease in emissivity of an unfinned tube. The ratio of heat transfer for an array having $\varepsilon = 0.8$ to that for a comparable array having $\varepsilon = 1.0$ varied from 0.93 for $N_R = 0.20$, $N_L = 0.25$, and $N_C = 0$; to 0.82 for $N_R = 0.67$, $N_L = 2.0$, and $N_C = 0.8$. Theoretically, the variation would cover the range from 0.8 to 1.0 as the fin spacing varied from zero to infinity. There is no case for which the addition of fins has a detrimental effect on heat transfer, assuming the fins and tube have the same emissivity. The decrease of Q^* with ε is nearly linear for $0.8 \leqq \varepsilon \leqq 1$ so that Q^* for any emissivity in that range may be obtained by linear interpolation of the $\varepsilon = 0.8$ and 1.0 results.

The effects of N_C, N_R, and N_L on Q^* may be seen in Figs. 5–8. Note that Q^* decreases sharply with increasing conduction number N_C so that the conductive resistance which is proportional to $1/kt_i$ must be kept relatively small. Since increased t_i implies an increase in system weight, a tradeoff situation arises in which the optimum value of t_i must be determined. Also note that Q^* increases with decreasing N_R and N_L. This corresponds to large, closely spaced fins. If the criterion for design is maximum heat transfer per unit weight, however, the best design requires

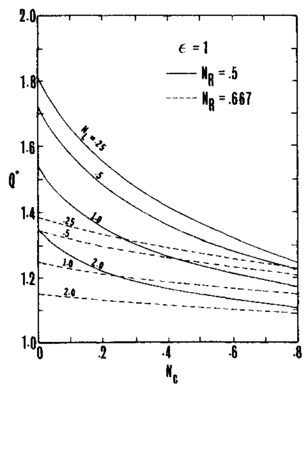

Fig. 6 Dimensionless heat flux from the fin array; $\varepsilon = 1.0$, $N_R = 0.5$ and 0.667.

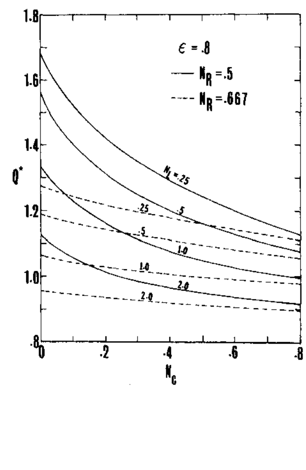

Fig. 8 Dimensionless heat flux from the fin array; $\varepsilon = 0.8$, $N_R = 0.5$, and 0.667.

Table 1 Results of calculations for example case

N_R	r_o in.	N_L	L in.	N_C	t_i mils	Vol in.^{3}
0.2	5	0.25	1.25	0.155	2.70	0.760
0.2	5	0.5	2.5	0.105	3.98	0.560
0.2	5	1.0	5.0	0.055	7.60	0.535
0.2	5	2.0	10.0	0.025	16.71	0.587
0.25	4	0.25	1.0	0.165	2.53	0.572
0.25	4	0.5	2.0	0.110	3.80	0.430
0.25	4	1.0	4.0	0.057	7.33	0.415
0.25	4	2.0	8.0	0.020	20.90	0.591
0.333	3	0.25	0.75	0.165	2.53	0.424
0.333	3	0.5	1.5	0.105	4.10	0.343
0.333	3	1.0	3.0	0.040	10.76	0.450
0.333	3	2.0	6.0	No solution →		
0.5	2	→		No solution →		
0.667	1.5	→		No solution →		

the smallest value of outside radius which can produce the required heat transfer, and there is an optimum fin-to-fin spacing which must be determined. An example problem illustrating this is presented in Appendix A.

The numerical method which has been described appears to have two advantages over other methods used to solve similar problems. These are its simplicity and efficiency. Separating the radiation and conduction analyses greatly simplifies the programing effort. The efficiency is due primarily to the fact that the matrix inversion, which is the most time-consuming operation, is done only once for each problem and is not repeated for each iteration of the temperature distribution. This concept is discussed in Ref. 14. Even though 10–40 iterations of the temperature distribution may be required, the calculation time is minimal. Therefore, a simple scheme to force convergence (the damping discussed earlier) can be used, rather than the more complex methods such as those described in Refs. 10 and 11. The efficiency is further improved by solving a series of cases in succession in which N_C is varied. Since the geometry and emissivities are the same for all of these cases, the entire series can be completed without repeating the matrix inversion. The IBM 7094 time averaged 0.49 sec for each problem, or less than 5 min for 480 cases.

Another numerical method which has been used involves combining the finite difference representations of the conduction and radiation equations into one system of nonlinear algebraic equations. Such a system may be solved by a Newton-Raphson procedure which would uniformly converge. This requires matrix inversion for each iteration, however, so that the computation time must be greater than that required in the method presented here.

Conclusions

A numerical method has been developed to predict the temperature distribution and heat-transfer rate for an array of circular fins of trapezoidal profile. The method is quite general since it includes radiative interactions among the elements of the array, variable thermal conductivity, gray surfaces, and variable profile area. The same computer program could be used to analyze other geometries by replacing the shape factor subroutine and changing a small number of geometric calculations in the main program. The numerical method is believed to be more efficient than other methods used to solve comparable problems.

Appendix A: Example Design Problem

Consider a tube having a radius of one inch with a surface temperature of 100°F. Aluminum circular fins of triangular cross section are to be added to produce a radiative heat transfer 1.8 times as large as that of the unfinned tube. Assume all surfaces have a coating which has an emissivity of 1.0.

A combination of values of N_R and N_L is selected. The results discussed in this report are then used to determine the corresponding N_C for $Q^* = 1.8$. Note that some combinations may not be possible. The definition of N_C is then used to compute t_i, the base thickness of the fin. The volume of fin material per unit length of tube can now be calculated. The results of these calculations for the case just defined are summarized in Table 1. Note that for any fixed value of N_R there is a particular value of N_L that minimizes the fin volume. The optimum design is $r_o = 3$ in., $L = 1.5$ in., and $t_i = 0.0041$ in.

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